

Traveling solitons in Lorentz-violating systems

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Abstract

In this work we present a class of traveling solitons in Lorentz-violating systems. In the case of Lorentz violating scenarios, it is usual to construct static solitonic configurations. Here it is shown that it is possible to construct some traveling solitons which, as it should be expected, can not be mapped into a static configuration by means of Lorentz boosts due to its explicit breaking. Furthermore, in the model studied, a complete set of solutions is obtained. The solutions present a critical behavior controlled by the choose of an arbitrary integration constant.

1 Introduction

The study of the problem of Lorentz symmetry breaking appeared in the physics literature motivated by the fact that the superstring theories suggest that Lorentz symmetry should be violated at higher energies [1]. Recently, a large amount of works considering the impact of some kind of Lorentz symmetry breaking have appeared in the literature. For instance, some years ago, Carrol, Field and Jackiw [2] addressed the problem with CPT (Charge conjugation-Parity-Time reversal) symmetry violation. On the other hand, some impact over the standard model due to Lorentz and CPT symmetries were discussed by Colladay and Kostelecky [3, 4]. Other problem analyzed in the literature is the spontaneous breaking of the four-dimensional Lorentz invariance of the QED [5]. At this point, it is interesting to mention that a space-time with torsion interacting with a Maxwell field by means of Chern-Simons-like term was introduced by the authors in Ref. [5]. In this case, it is possible to explain the optical activity in the synchrotron radiation emitted by cosmological distant radio sources.

Recently motivated by the problem of Lorentz symmetry violating gauge theories in connection with gravity models, Boldo *et al.* [6] have analyzed the graviton excitations and

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Lorentz-violating gravity with cosmological constant. It is important to remark that a considerable effort has been done experimentally to observe signs of the Lorentz and CPT symmetries violation effects. In fact, in a very recent work, Maccione, Liberati and Sigh [7] have shown that experimental data on the photon content of ultrahigh-energy cosmic rays lead to strong constraints over the Lorentz symmetry violations in stringy space-time foam models. This was done by studying the time delay between γ rays of different energies from extragalactic sources. Moreover, Giulia Gubitosi *et al.* [8] have introduced an important role in the study of Planck-scale modifications to electrodynamics characterized by a space-like symmetry-breaking vector. This year, several studies involving Lorentz violation has appeared in the literature [6]-[16].

Finally, it is important to remark that nonlinear models which have topological solutions are very interesting and important in many branches of physics [17]-[20]. In a recent work [21] it was shown that some nonlinear models in two-dimensional space-time were two scalar fields interact in the Lorentz and CPT violating scenarios present static solitonic configurations. This was done by generalizing a model presented by Barreto and collaborators [22]. Finally, in a very recent work, Bazeia *et al.* [26] also have analyzed the effects of the Lorentz violation on topological defects generated by two real scalar fields. In that case, the Lorentz-violating is induced by a fixed tensor coefficient that couples the two fields. In all of these examples, the presented solitonic configurations were static. In this work we are going to show that it is possible to find nontrivial traveling solutions in this kind of scenario. This is going to be done by taking as example a generalization of some models recently discussed in the literature [21]-[26]. As a consequence, we present a class of traveling solitons in Lorentz-violating systems as well as some static configurations. Finally it is shown that the static configurations are not the limit of the traveling ones. This is done by using an approach developed to deal with some classes of nonlinear models in two-dimensional space-time of two interacting scalar fields which were presented in [27]. In this last reference it was shown that these systems in $1 + 1$ dimensions, the so-called orbit equation can be cast in a form of linear first-order differential equation, thus leading to general solutions of the system. We also show that the solutions present a critical behavior controlled by the choose of an arbitrary integration constant.

2 The model

Some years ago, it was presented in [21] a two-field model in $1+1$ dimensions where the Lorentz breaking Lagrangian density generalizes some results in the literature. That Lagrangian density contains vector functions with dependence on the dynamical scalar fields. Moreover, the mentioned vector functions are responsible by the Lorentz symmetry breaking. On the other hand, in reference [26], the effects of the Lorentz violation on topological defects generated by two real scalar fields was analyzed too, in this last one the Lagrangian density has a tensor which it is the term which breaks the Lorentz symmetry. Thus, in this work we construct a generalized two-field model in $1+1$ dimensions which is described by the

Lagrangian density

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - G^\mu(\phi, \chi)\partial_\mu\phi - F^\mu(\phi, \chi)\partial_\mu\chi + \\ & -\gamma k^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) - pk^{\mu\nu}\partial_\mu\phi\partial_\nu\chi - V(\phi, \chi),\end{aligned}\quad (1)$$

where $\mu = 0, 1$, $G^\mu(\phi, \chi)$ and $F^\mu(\phi, \chi)$ are vector functions, and $V(\phi, \chi)$ is the potential. Furthermore, $k^{\mu\nu}$ is a constant tensor, here represented by a 2×2 matrix, where $\alpha_1, \alpha_2, \alpha_3$ and α_4 are arbitrary parameters. In fact, a similar process of breaking the Lorentz symmetry was put forward by Anacleto *et al.* [16] in a recent work, where the tensor $k^{\mu\nu}$ is a 4×4 matrix, in that case the authors studied the problem of the acoustic black holes from Abelian Higgs model with Lorentz symmetry breaking. Here, as advertised, the tensor $k^{\mu\nu}$ is written as

$$k^{\mu\nu} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{pmatrix}. \quad (2)$$

Note that, from the Lagrangian density (1), we can recover the one presented in the work by Bazeia *et al* [26] by choosing $\gamma = 0$, $G^0(\phi, \chi) = F^0(\phi, \chi) = 0$, $\alpha_1 = \alpha_4 = \beta$, $\alpha_2 = \alpha_3 = \alpha$ and $p = -1$. Furthermore, we can also recover the lagrangian density presented in [21, 22] by setting conveniently the above defined parameters. Therefore, we have a more general model including vector functions and a constant tensor. It is important to remark that the more general model presented here, can be used to bring more information about the impact of the Lorentz violation of important systems like, for instance, those presenting topological structures [21, 22].

From the Lagrangian density (1), we can to write the corresponding equations of motion

$$(1 - \gamma \alpha_1)\ddot{\phi} - (1 + \gamma \alpha_4)\phi'' - p(\alpha_1\ddot{\chi} + \alpha_4\chi'') + (F_\phi^0 - G_\chi^0)\dot{\chi} + (F_\phi^1 - G_\chi^1)\chi' + \quad (3)$$

$$-(\alpha_3 + \alpha_2)(\gamma\dot{\phi}' + p\dot{\chi}') + V_\phi = 0,$$

$$(1 - \gamma \alpha_1)\ddot{\chi} - (1 + \gamma \alpha_4)\chi'' - p(\alpha_1\ddot{\phi} + \alpha_4\phi'') - (F_\phi^0 - G_\chi^0)\dot{\phi} - (F_\phi^1 - G_\chi^1)\phi' + \quad (4)$$

$$-(\alpha_3 + \alpha_2)(\gamma\dot{\chi}' + p\dot{\phi}') + V_\chi = 0,$$

where the dot stands for derivative with respect to time, while the prime represents derivative with respect to x , $V_\phi \equiv \partial V / \partial \phi$ and $V_\chi \equiv \partial V / \partial \chi$. It can be seen that the two equations in above are carrying informations of the Lorentz breaking of the model through the presence of the α_i parameters and the vector functions. But, as a consequence of the model studied in this work, in general we can not solve analytically the above differential equations. However one can consider an interesting case for the fields configurations, where one searches for traveling waves solutions. Configurations that exhibit traveling waves has an important impact when we study boundary states for D-branes and the supergravity fields for a D-brane [23]-[25].

Then, let us begin our search for traveling waves solutions in the form $\phi = \phi(u)$ and

$\chi = \chi(u)$ with $u = Ax + Bt$. Thus, the equations (3) and (4) take the form

$$-\phi_{uu} + \tilde{\beta}\chi_{uu} - \tilde{\alpha}\chi_u + \tilde{V}_\phi = 0, \quad (5)$$

$$-\chi_{uu} + \tilde{\beta}\phi_{uu} + \tilde{\alpha}\phi_u + \tilde{V}_\chi = 0, \quad (6)$$

with the definitions

$$\tilde{\beta} \equiv -\frac{p[(\alpha_2 + \alpha_3)AB + \alpha_4A^2 + \alpha_1B^2]}{(1 + \gamma\alpha_4)A^2 - (1 - \gamma\alpha_1)B^2 + AB\gamma(\alpha_2 + \alpha_4)}, \quad (7)$$

$$\tilde{\alpha} \equiv -\frac{B(F_\phi^0 - G_\chi^0) + A(F_\phi^1 - G_\chi^1)}{(1 + \gamma\alpha_4)A^2 - (1 - \gamma\alpha_1)B^2 + AB\gamma(\alpha_2 + \alpha_4)}, \quad (8)$$

$$\tilde{V}_\phi \equiv \frac{V_\phi}{(1 + \gamma\alpha_4)A^2 - (1 - \gamma\alpha_1)B^2 + AB\gamma(\alpha_2 + \alpha_4)}, \quad (9)$$

$$\tilde{V}_\chi \equiv \frac{V_\chi}{(1 + \gamma\alpha_4)A^2 - (1 - \gamma\alpha_1)B^2 + AB\gamma(\alpha_2 + \alpha_4)}. \quad (10)$$

In order to decouple the pair of second order differential equations, we multiply the equation (5) by ϕ_u and the equation (6) by χ_u . Thus, it is not difficult to conclude that, after adding the two equations, one can write

$$\frac{d}{du} \left[-\frac{1}{2}(\phi_u^2 + \chi_u^2) + \tilde{\beta}\phi_u\chi_u + \tilde{V}(\phi, \chi) \right] = 0. \quad (11)$$

In this case, we have

$$-\frac{1}{2}(\phi_u^2 + \chi_u^2) + \tilde{\beta}\phi_u\chi_u + \tilde{V}(\phi, \chi) = c_0. \quad (12)$$

The above equation can be rewritten for the case where $c_0 = 0$ which is necessary in order to allow solitonic solutions, otherwise one obtains oscillating or complex solutions [?]. Therefore, we get

$$-\frac{1}{2}(\phi_u^2 + \chi_u^2) + \tilde{\beta}\phi_u\chi_u + \tilde{V}(\phi, \chi) = 0. \quad (13)$$

Note that in the above equation, the dependence in $\tilde{\alpha}$ has disappeared. However, the dependence of the system in terms of the Lorentz breaking parameters is still present but it is implicit. Now, in order to desacouple the above equation, we apply the rotation

$$\begin{pmatrix} \phi(u) \\ \chi(u) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \theta(u) \\ \varphi(u) \end{pmatrix}, \quad (14)$$

thus, the equation (13) is rewritten as

$$-\frac{1}{2}(1 - \tilde{\beta})\theta_u^2 - \frac{1}{2}(1 + \tilde{\beta})\varphi_u^2 + \tilde{V}(\theta, \varphi) = 0. \quad (15)$$

Furthermore, performing the dilations

$$\theta(u) = \frac{\sigma(u)}{\sqrt{1 - \tilde{\beta}}}, \quad \varphi(u) = \frac{\rho(u)}{\sqrt{1 + \tilde{\beta}}}, \quad (16)$$

one gets

$$-\frac{1}{2}\sigma_u^2 - \frac{1}{2}\rho_u^2 + \tilde{V}(\sigma, \rho) = 0. \quad (17)$$

At this point one can verify that the above equation allow one to write two first-order coupled differential equations. In this case it is usual to impose that the potential must be written in terms of a superpotential like

$$\bar{V}(\sigma, \rho) = \frac{1}{2} \left(\frac{\partial W(\sigma, \rho)}{\partial \sigma} \right)^2 + \frac{1}{2} \left(\frac{\partial W(\sigma, \rho)}{\partial \rho} \right)^2, \quad (18)$$

which leads to the following set of equations

$$\frac{d\sigma}{du} = \pm W_\sigma, \quad \frac{d\rho}{du} = \pm W_\rho, \quad (19)$$

where $W_\sigma \equiv \partial W(\sigma, \rho)/\partial \sigma$ and $W_\rho \equiv \partial W(\sigma, \rho)/\partial \rho$, and this will leads us to the solitonic solutions we are looking for.

In order to analyze the energy of the configurations obtained, we write the energy-momentum tensor in the form

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} \partial^\nu \chi - g^{\mu\nu} \mathcal{L}. \quad (20)$$

Therefore, the energy density for the lagrangian density (1) is given by

$$\begin{aligned} T^{00} = & \frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} + \left(\frac{1}{2} + \gamma\alpha_4 \right) (\phi'^2 + \chi'^2) + G^1(\phi, \chi)\phi' + F^1(\phi, \chi)\chi' + \\ & -p\alpha_1\dot{\phi}\dot{\chi} + \gamma(\alpha_2 + \alpha_3)\phi'\dot{\phi} + \gamma(\alpha_2 + \alpha_3)\chi'\dot{\chi} + p\alpha_4\phi'\chi' + p\alpha_2\dot{\phi}\chi' + \\ & + p\alpha_3\phi'\dot{\chi} + V(\phi, \chi). \end{aligned} \quad (21)$$

For the traveling waves solutions, the energy density is written in the form

$$\begin{aligned} T_{traveling}^{00} = & \left[\frac{B^2}{2} + \left(\frac{1}{2} + \gamma\alpha_4 \right) A^2 + \gamma(\alpha_2 + \alpha_3)AB \right] \phi_u^2 \\ & + \left[\frac{B^2}{2} + \left(\frac{1}{2} + \gamma\alpha_4 \right) A^2 + \gamma(\alpha_2 + \alpha_3)AB \right] \chi_u^2 + p [\alpha_4 A^2 - \alpha_1 B^2 + (\alpha_2 + \alpha_3)AB] \phi_u \chi_u \\ & + A[G^1(\phi, \chi)\phi_u + F^1(\phi, \chi)\chi_u] + V(\phi, \chi). \end{aligned} \quad (22)$$

Now, we choose the superpotential which was used in [27], which is written as

$$W(\sigma, \rho) = -\lambda\sigma + \frac{\lambda}{3}\sigma^3 + \mu\sigma\rho^2. \quad (23)$$

In this case, the solutions presented in Ref. [27], with $\lambda = \mu$, are given by

$$\sigma_+(u) = \frac{(c_0^2 - 4)e^{4\mu(u-u_0)} - 1}{[c_0e^{2\mu(u-u_0)} - 1]^2 - 4e^{4\mu(u-u_0)}}, \sigma_-(u) = \frac{4 - c_0^2 + e^{4\mu(u-u_0)}}{[e^{2\mu(u-u_0)} - c_0]^2 - 4}, \quad (24)$$

$$\rho_+(u) = \frac{4e^{2\mu(u-u_0)}}{[c_0e^{2\mu(u-u_0)} - 1]^2 - 4e^{4\mu(u-u_0)}}, \rho_-(u) = \frac{4e^{2\mu(u-u_0)}}{[e^{2\mu(u-u_0)} - c_0]^2 - 4},$$

where we must impose that $c_0 \leq -2$ in both solutions. On the other hand, in the case where $\lambda = 4\mu$, the exact solutions are written as

$$\sigma_+(u) = \frac{4 + (16c_0 - 1)e^{8\mu(u-u_0)}}{[2 + e^{4\mu(u-u_0)}]^2 - 16c_0e^{8\mu(u-u_0)}}, \sigma_-(u) = \frac{16c_0 + 4e^{8\mu(u-u_0)} - 1}{[1 + 2e^{4\mu(u-u_0)}]^2 - 16c_0}, \quad (25)$$

$$\rho_+(u) = -\frac{2e^{2\mu(u-u_0)}}{\sqrt{[(1/2)e^{4\mu(u-u_0)} + 1]^2 - 4c_0e^{8\mu(u-u_0)}}}, \rho_-(u) = -\frac{4e^{2\mu(u-u_0)}}{\sqrt{[1 + 2e^{4\mu(u-u_0)}]^2 - 16c_0}}.$$

In this case, we impose that $c_0 \leq 1/16$. It is important to remark that making the exchange of $\sigma \rightarrow \rho$ and $\rho \rightarrow \sigma$ in the case where $\lambda = \mu$, the equation of motion (17) remains invariant. Thus, the solutions where kinks become lumps and *vice-versa* shall appear, and this is used in order to generate the complete set of orbits appearing in the Figure 4. In fact, this symmetry is important for the generation of the complete set of orbits connecting the vacua.

Thus, the fields $\phi(u)$ and $\chi(u)$ are given by

$$\phi_{\pm}(u) = \frac{1}{\sqrt{2}} \left[\frac{\sigma_{\pm}(u)}{\sqrt{1 - \tilde{\beta}}} - \frac{\rho_{\pm}(u)}{\sqrt{1 + \tilde{\beta}}} \right], \quad (26)$$

$$\chi_{\pm}(u) = \frac{1}{\sqrt{2}} \left[\frac{\sigma_{\pm}(u)}{\sqrt{1 - \tilde{\beta}}} + \frac{\rho_{\pm}(u)}{\sqrt{1 + \tilde{\beta}}} \right].$$

Now, using the solutions presented in the reference [27] which are represented here by (24) and (25), we have the complete set of solutions with position and time dependence.

Here, we call attention to the fact that the static solutions for the equations (3) and (4) are different from the traveling wave ones. This difference can be seen from an inspection of the static fields differential equations

$$-\phi'' + \bar{\beta}\chi'' + \bar{\alpha}\chi' + \bar{V}_{\phi} = 0, \quad (27)$$

$$-\chi'' + \bar{\beta}\phi'' - \bar{\alpha}\phi' + \bar{V}_{\chi} = 0, \quad (28)$$

where now, one have

$$\bar{\beta} \equiv \frac{-p\alpha_4}{(1+\gamma\alpha_4)}, \bar{\alpha} \equiv \frac{(F_\phi^1 - G_\chi^1)}{(1+\gamma\alpha_4)}, \quad (29)$$

$$\bar{V}_\phi \equiv \frac{V_\phi}{(1+\gamma\alpha_4)} \quad \text{and} \quad \bar{V}_\chi \equiv \frac{V_\chi}{(1+\gamma\alpha_4)}.$$

In particular, if $\gamma = 0$, $G^\mu(\phi, \chi) = F^\mu(\phi, \chi) = 0$, $\alpha_1 = \alpha_4 = \beta$, $\alpha_2 = \alpha_3 = \alpha$ and $p = -1$, ones recovers the model presented by Bazeia *et al* [26]. In the static case, the equations of motion presented by the authors are given by

$$-\phi'' + \beta\chi'' + V_\phi = 0, \quad (30)$$

$$-\chi'' + \beta\phi'' + V_\chi = 0. \quad (31)$$

Here, it is interesting to note that the pair of equations presented in [26] for the static solutions takes on a different form compared with the equations (27) and (28). In fact, we can recover the equations of motion presented in the work of Bazeia [26] by setting the correct parameters. But the general static configurations are given by equations (27) and (28), which are carrying more informations of the terms of the Lorentz breaking of the model.

The difference discussed above can be seen in Figures 1 and 2. In Figure 3 the complete set of classes of orbits are illustrated.

3 Conclusions

In this work we have shown that a class of traveling solitons in Lorentz-violating systems can be analytically obtained, which happens despite the fact that there is no Lorentz symmetry and consequently one can not recover the traveling solutions from the static one, just performing Lorentz boosts. This has been done by using nonlinear models in two-dimensional space-time of two interacting scalar fields which were presented in [27]. Furthermore, in the model studied, a complete set of solutions was obtained. The solutions present a critical behavior controlled by the choose of an arbitrary integration constant.

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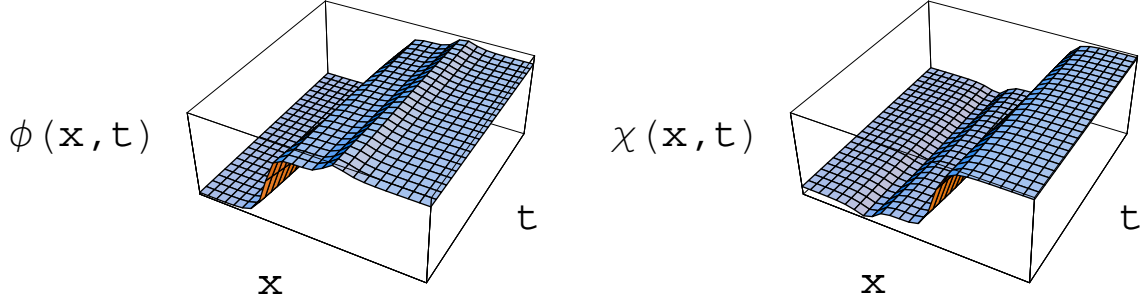


Figure 1: Solution $\phi(x, t)$ and $\chi(x, t)$, with position and time dependence for $\lambda = 4\mu$, $\mu = 0.5$, $c_0 = 1/16.0001$, $A = 0.5$, $B = -0.1$ and $\beta = 0.7$.

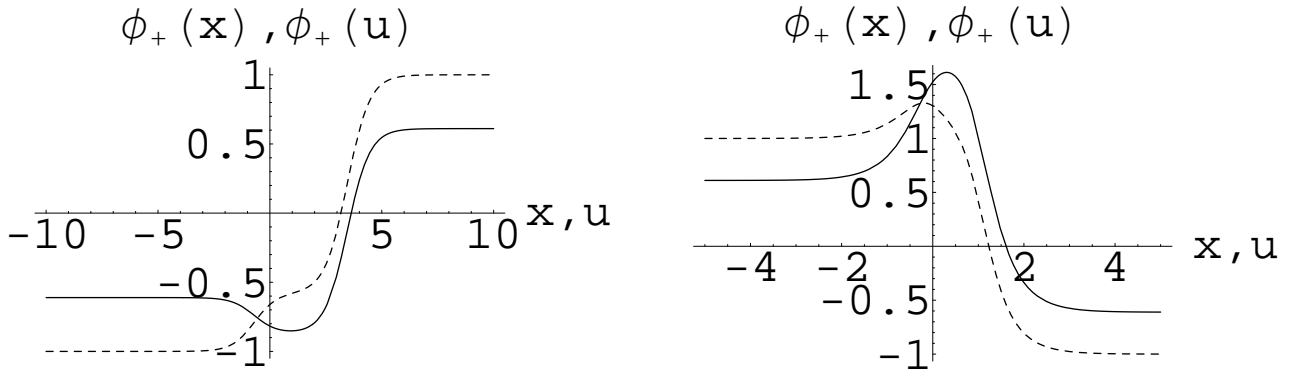


Figure 2: Traveling solitons solutions and static solutions for $\lambda = \mu = 1$ and $c_0 = -2.001$. The dashed line corresponds to the static case with $\beta = 0.5$, $A = 1$. The thin continuous line corresponds to the traveling wave case for $\beta = 0.5$, $\alpha = 0.4$, $A = 1$, $B = -1.5$.

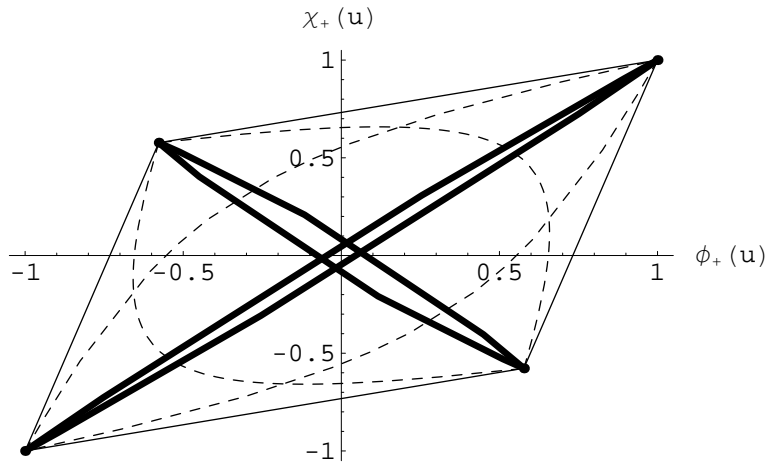


Figure 3: Orbit for the solutions with $\lambda = \mu = 1$ and $\tilde{\beta} = 0.5$. The thin line corresponds to the case where $c_0 = -2.00001$, the dash line corresponds to the case where $c_0 = -2.4$ and the thick line with $c_0 = -20$.